

# AP Calculus AB Summer Review Packet

Simplify

$$1. \frac{x^3 - 9x}{x^2 - 7x + 12}$$

$$2. \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$4. \frac{9 - x^{-2}}{3 - x^{-1}}$$

Rationalize the denominator

$$5. \frac{2}{\sqrt{3} + \sqrt{2}}$$

$$6. \frac{4}{1 - \sqrt{5}}$$

$$7. \frac{1 - \sqrt{5}}{1 + \sqrt{3}}$$

Write each of the following expressions in the form of  $ca^p b^q$  where  $c$ ,  $p$ , and  $q$  are numbers

$$8. \frac{(2a^2)^3}{b}$$

$$10. \frac{a(2/b)}{3/a}$$

$$12. \frac{a^{-1}}{(b^{-1})\sqrt{a}}$$

$$9. \sqrt{9ab^3}$$

$$11. \frac{ab - a}{b^2 - b}$$

$$13. \left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$$

Solve for  $x$ . Do not use a calculator

$$14. 5^{(x+1)} = 25$$

$$16. \log_2 x = 3$$

$$15. \frac{1}{3} = 3^{2x+2}$$

$$17. \log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$$

Simplify

$$18. \log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1)$$

$$19. 2 \log_4 9 - \log_2 3$$

$$20. 3^{2 \log_3 5}$$

Simplify

$$21. \log_{10} 10^{1/2}$$

$$22. \log_{10} \frac{1}{10^x}$$

$$23. 2 \log_{10} \sqrt{x} + 3 \log_{10} x^{1/3}$$

## AP Calculus AB Summer Review Packet

Welcome to AP Calculus AB. This packet is a set of problems that you should be able to do before entering this course. You are not required to do EVERY problem in this packet, but you are responsible for all content. I will not collect these problems, but you will be tested on these questions during the first week of school. I'll give you a few days to ask me questions and then you'll have a test on this background material. Make sure you circle any questions that you are unsure about so that you can get clarification during the Q & A period before the test is given.

There are numerous resources available to you on my web site at <http://fightingvikings.com/Math/Index.html>. Under the Miscellaneous tab, you'll find formulas from previous courses along with unit circle values and other reference material. I would also recommend reviewing instructional videos on any topics that you may find unclear. Again, my web site has numerous links to additional online instructional videos such as Khan Academy and Wolfram Alpha. At the end of the problems, I've included an answer key for you to do some self-check. (You're the first group to use this packet, so if you find an error, please email me at [james\\_m\\_kuhn@mcpsmd.org](mailto:james_m_kuhn@mcpsmd.org) so that I can correct it. )

Probably the biggest difficulty for students is the limitations placed on the use of a calculator. Approximately 70% of the AP exam is without a calculator, so that is the way the course is taught. Approximately 70% of your evaluations will be without a calculator. (That includes a four function calculator, no calculator is allowed at all). When tested on this material, it will be without a calculator.

Finally, I cannot stress enough the importance of these background skills. Calculus is easy, it's the algebra that is hard. Most of the time, you'll understand the calculus concept being taught, but will struggle to get the correct answer because of your background skills. Please be diligent and figure out what you need help with so that I can clarify for you. A little extra work this summer will go a long way to help you succeed in the upcoming year.

I look forward to having you in class and working together to accomplish your goals. I think you'll find this course challenging but enjoyable at the same time. Should you have any questions throughout the summer, please feel free to contact me at [james\\_m\\_kuhn@mcpsmd.org](mailto:james_m_kuhn@mcpsmd.org).

Hope you have a great summer!

Mr. Kuhn

Given the graph of  $y = \sin x$ , sketch the graphs of:

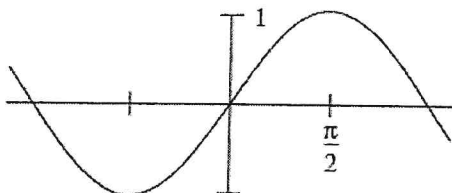
51.  $\sin\left(x - \frac{\pi}{4}\right)$

52.  $\sin\left(\frac{\pi}{2}\right)$

53.  $2 \sin x$

54.  $\cos x$

55.  $\frac{1}{\sin x}$



Solve the equations

56.  $4x^2 + 12x + 3 = 0$

58.  $\frac{x+1}{x} - \frac{x}{x+1} = 0$

57.  $2x + 1 = \frac{5}{x+2}$

Find the remainders on division of

59.  $x^5 - 4x^4 + x^3 - 7x + 1$  by  $x + 2$

60.  $x^5 - x^4 + x^3 + 2x^2 - x + 4$  by  $x^3 + 1$

61. The equation  $12x^3 - 23x^2 - 3x + 2 = 0$  has a solution  $x = 2$ . Find all other solutions.

62. Solve for  $x$ , the equation  $12x^3 + 8x^2 - x - 1 = 0$  (all solutions are rational and between  $\pm 1$ )

Solve the inequalities. Give the solution in interval notation

63.  $x^2 + 2x - 3 \leq 0$

64.  $\frac{2x-1}{3x-2} \leq 1$

65.  $\frac{2}{2x+3} > \frac{2}{x-5}$

Solve for  $x$ . Give the solution in interval notation

66.  $|-x + 4| \leq 1$

67.  $|5x - 2| = 8$

68.  $|2x + 1| > 3$

Solve the following equations for the indicated variable

$$24. \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \text{ for } a$$

$$25. V = 2(ab + bc + ca), \text{ for } a$$

$$26. A = 2\pi r^2 + 2\pi rh, \text{ for positive } h$$

$$27. A = P + \pi rP, \text{ for } P$$

$$28. 2x - 2yd = y + xd, \text{ for } d$$

$$29. \frac{2x}{4\pi} + \frac{1-x}{2} = 0, \text{ for } x$$

For each equation complete the square and reduce to one of the standard forms  $y - y_1 = A(x - x_1)^2$  or  $x - x_1 = (y - y_1)^2$

$$30. y = x^2 + 4x + 3$$

$$31. 3x^2 + 3x + 2y = 0$$

$$32. 9y^2 - 6y - 9 - x = 0$$

Factor completely

$$33. x^6 - 16x^4$$

$$34. 4x^3 - 8x^2 - 25x + 50$$

$$35. 8x^3 + 27$$

$$36. x^4 - 1$$

Find all real solutions

$$37. x^6 - 16x^4 = 0$$

$$38. 4x^3 - 8x^2 - 25x + 50 = 0$$

$$39. 8x^3 + 27 = 0$$

Solve for  $x$

$$40. 3\sin^2 x = \cos^2 x; \quad 0 \leq x < 2\pi$$

$$41. \cos^2 x - \sin^2 x = \sin x; \quad -\pi < x \leq \pi$$

$$42. \tan x + \sec x = 2 \cos x; \quad -\infty < x < \infty$$

Without using a calculator, evaluate the following:

$$43. \cos 210^\circ$$

$$44. \sin \frac{5\pi}{4}$$

$$45. \tan^{-1}(-1)$$

$$46. \sin^{-1}(-1)$$

$$47. \cos \frac{9\pi}{4}$$

$$48. \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$49. \tan\left(\frac{7\pi}{6}\right)$$

$$50. \cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$$

Determine the equation of the following lines

69. The line through  $(-1, 3)$  and  $(2, -4)$
70. The line through  $(-1, 2)$  and perpendicular to the line  $2x - 3y + 5 = 0$
71. The line through  $(2, 3)$  and the midpoint of the line segment from  $(-1, 4)$  to  $(3, 2)$
72. Find the point of intersection of the lines:  $3x - y - 7 = 0$  and  $x + 5y + 3 = 0$
73. Shade the region in the  $xy$ -plane that is described by the inequalities  $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$

Find the equations of the following circles:

74. The circle with center at  $(1, 2)$  that passes through the point  $(-2, -1)$
75. The circle that passes through the origin and has intercepts equal to 1 and 2 on the  $x$  and  $y$  axes respectively.
76. For the circle  $x^2 + y^2 + 6x - 4y + 3 = 0$  find the center and the radius
77. Find the domain of  $\frac{3x+1}{\sqrt{x^2+x-2}}$

Find the domain and range of:

78.  $f(x) = 7$

79.  $g(x) = \frac{5x-3}{2x+1}$

80.  $f(x) = \frac{|x|}{x}$

Simplify  $\frac{f(x+h)-f(x)}{h}$  when

81.  $f(x) = 2x + 3$

82.  $f(x) = \frac{1}{x+1}$

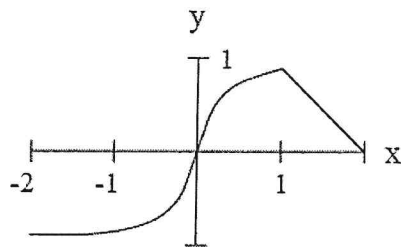
83.  $f(x) = 3x^2 - x + 5$

The graph of the functions  $y = f(x)$  is given as follows: Determine the graphs of the functions:

84.  $f(x + 1)$

85.  $f(-x)$

86.  $|f(x)|$



Sketch the graphs of the functions

87.  $g(x) = |3x + 2|$

88.  $h(x) = |x(x - 1)|$

89. The graph of a quadratic function has x-intercepts  $-1$  and  $3$  and a range consisting of all numbers less than or equal to  $4$ . Determine an expression for the function.

90. Sketch the graph of the quadratic function  $y = 2x^2 - 4x + 3$

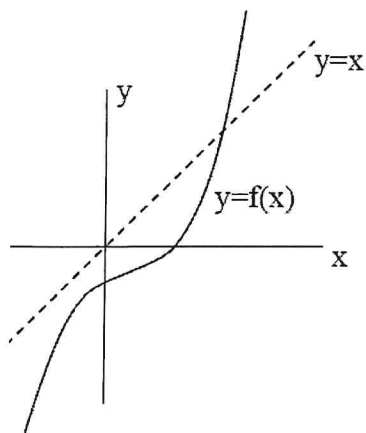
Find the inverse of the functions

91.  $f(x) = 2x + 3$

93.  $f(x) = x^2 - 2x - 1, x > 0$

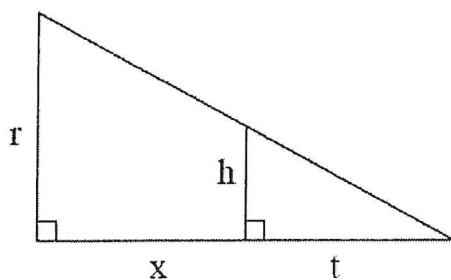
92.  $f(x) = \frac{x+2}{5x-1}$

94. A function  $f(x)$  has the graph below. Sketch the graph of the inverse function  $f^{-1}(x)$ .

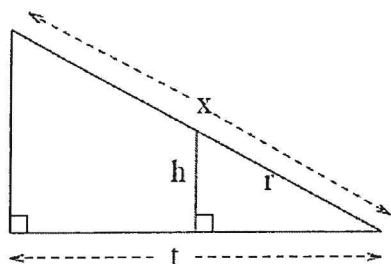


For problems 96 and 97, express  $x$  in terms of the other variables in the picture:

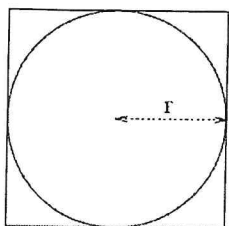
95.



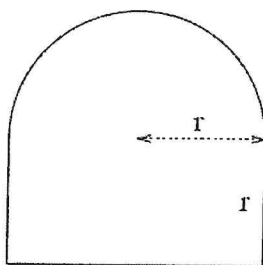
96.



97. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below



98. Find the formula for the perimeter of the window of the shape in the picture below



99. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is  $10\text{ m}$  high and has a radius of  $3\text{ m}$  as the top. If the water is  $5\text{ m}$  deep (in the middle) what is the surface area of the top of the water?
100. Two cars start moving from the same point. One travels south at  $100\text{ km/hr}$ , the other west at  $50\text{ km/hr}$ . How far apart are they two hours later?
101. A kite is  $100\text{ m}$  above the ground. If there are  $200\text{ m}$  of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

If  $f(x) = 2x - 3$  and  $g(x) = \sqrt{3x - 1}$ , Find:

102.  $f(g(x))$

103.  $g(f(x))$

104. If  $f(x) = \frac{3}{x}$  and  $g(x) = \frac{x}{2x-1}$ , Find  $f(g(x))$  and state its domain.

Decompose each composition function into individual function. (If  $y = f(u)$ , identify  $u$  and rewrite  $y$  in terms of  $u$ )

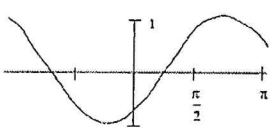
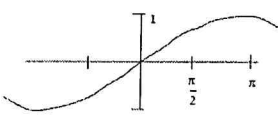
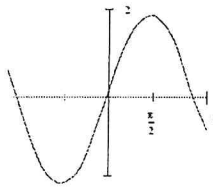
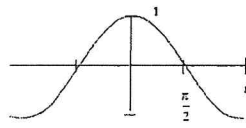
105.  $y = \sin 3x$

107.  $y = (x^2 - 2x + 5)^5$

106.  $y = \sqrt[5]{2x + 1}$

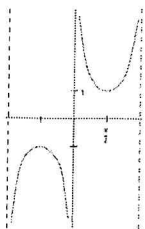
108.  $y = \cos^2 x$

## Answers

1.  $\frac{x^2+3x}{x-4}$
2.  $\frac{x^2-x}{5x}$
3.  $\frac{x+5}{3x+1}$
4.  $\frac{x}{x}$
5.  $2(\sqrt{3}-\sqrt{2})$
6.  $-1-\sqrt{5}$
7.  $\frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{-2}$
8.  $8a^6b^{-1}$
9.  $3a^{1/2}b^{3/2}$
10.  $\frac{2}{3}a^2b^{-1}$
11.  $ab^{-1}$
12.  $a^{-3/2}b$
13.  $a^{5/6}b^{1/2}$
14. 1
15.  $-\frac{3}{2}$
16. 8
17.  $\pm \frac{4}{25}$
18.  $\log_2(5(x+1))$
19.  $\log_2 3$
20. 25
21.  $\frac{1}{2}$
22.  $-x$
23.  $2 \log_{10} x$
24.  $\frac{bcx}{bc-cy-bz}$
25.  $\frac{V-2bc}{2(b+c)}$
26.  $\frac{A-2\pi r^2}{2\pi r}$
27.  $\frac{1+\pi r}{2x-y}$
28.  $\frac{x+2y}{\pi}$
29.  $\frac{\pi}{\pi-1}$
30.  $y+1=(x+2)^2$
31.  $y-\frac{3}{8}=-\frac{3}{2}\left(x+\frac{1}{2}\right)^2$
32.  $x+10=9\left(y-\frac{1}{3}\right)^2$
33.  $x^4(x-4)(x+4)$
34.  $(x-2)(2x-5)(2x+5)$
35.  $(2x+3)(4x^2-6x+9)$
36.  $(x-1)(x+1)(x^2+1)$
37.  $0, \pm 4$
38.  $2, \pm \frac{5}{2}$
39.  $-\frac{3}{2}$
40.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
41.  $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
42.  $\frac{\pi}{6} + 2k\pi$  and  $\frac{5\pi}{6} + 2k\pi$  where  $k \in I$
43.  $-\frac{\sqrt{3}}{2}$
44.  $-\frac{\sqrt{2}}{2}$
45.  $-\frac{\pi^2}{4}$
46.  $-\frac{\pi}{2}$
47.  $\frac{\sqrt{2}}{2}$
48.  $\frac{\pi^2}{3}$
49.  $\frac{\sqrt{3}}{3}$
50.  $\frac{3\pi}{4}$
51. 
52. 
53. 
54. 



55.



56.  $\frac{-3 \pm \sqrt{6}}{2}$

57.  $\frac{1}{2}$  or  $-3$

58.  $-\frac{1}{2}$

59.  $-89$

60.  $x^2 + 3$

61.  $-\frac{1}{3}$  or  $\frac{1}{4}$

62.  $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{2}$

63.  $[-3, 1]$

64.  $(-\infty, \frac{2}{3}) \cup [1, \infty)$

65.  $(-\infty, -8) \cup (-\frac{3}{2}, 5)$

66.  $[3, 5]$

67.  $2$  and  $-\frac{6}{5}$

68.  $(-\infty, -2) \cup (1, \infty)$

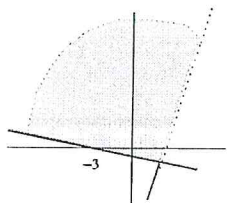
69.  $7x + 3y = 2$

70.  $3x + 2y = 1$

71.  $y = 3$

72.  $(2, -1)$

73.



74.  $(x - 1)^2 + (y - 2)^2 = 18$

75.  $(x - \frac{1}{2})^2 + (y - 1)^2 = \frac{5}{4}$

76. Center =  $(-3, 2)$ , radius =  $\sqrt{10}$

77.  $(-\infty, -2) \cup (1, \infty)$

78. Domain  $(-\infty, \infty)$  Range  $\{7\}$

79. Domain  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

Range  $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

80. Domain  $(-\infty, 0) \cup (0, \infty)$

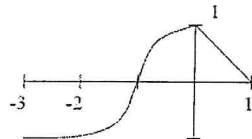
Range  $\{-1, 1\}$

81.  $2$

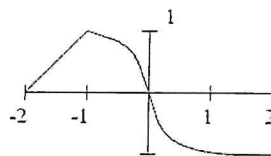
82.  $\frac{-1}{(x+1)(x+h+1)}$

83.  $6x + 3h - 1$

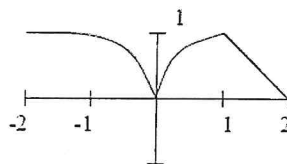
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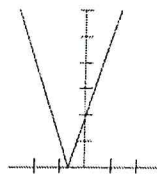
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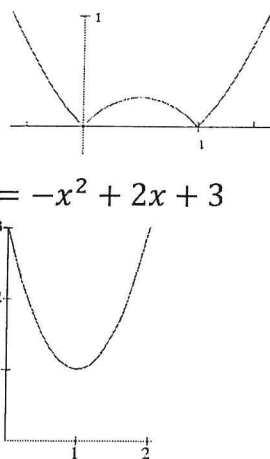
86.



87.

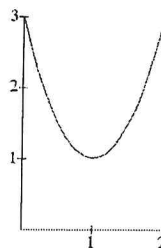


88.



89.  $y = -x^2 + 2x + 3$

90.

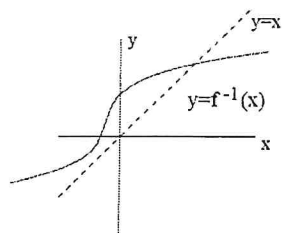


91.  $f^{-1} = \frac{x-3}{2}$

92.  $f^{-1} = \frac{x+2}{5x-1}$

93.  $f^{-1} = 1 + \sqrt{x+2}$  for  $x > -1$

94.



95.  $x = t \left( \frac{r-h}{h} \right)$

96.  $x = \frac{rt}{\sqrt{r^2-h^2}}$

97.  $1 - \frac{\pi}{4}$

98.  $4r + \pi r$

99.  $\frac{9\pi}{4}$

100.  $100\sqrt{5} \text{ KM}$

101.  $\frac{\pi}{6}$

102.  $2\sqrt{3x-1} - 3$

103.  $\sqrt{6x-10}$

104.  $\frac{6x-3}{x}$

Domain  $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

105. Let  $u = 3x$ , then  $y = \sin u$

106. Let  $u = 2x + 1$ , then  $y = \sqrt[5]{u}$

107. Let  $u = x^2 - 2x + 5$ ,  
then  $y = u^5$

108. Let  $u = \cos x$ , then  $y = u^2$

7. If  $f$  and  $g$  are two functions such that when composed in either order, the result is the identity function then  $f$  and  $g$  are inverses of each other. If  $f(x) = x^2$ ,  $x \leq 0$  and  $g(x) = -\sqrt{x}$ , are ' $f$ ' and ' $g$ ' inverses? Show that they are or are not inverses analytically via the above definition. Be SURE the details are clear!

8. If  $f(x) = x^2$ , find TWO functions,  $g$ , for which  $(f \circ g)(x) = 4x^2 - 12x + 9$ .

9. If:  $f(x) = \{(3,5), (2,4), (1,7)\}$ ,  $g(x) = \sqrt{x-3}$ ,  $h(x) = \{(3,2), (4,3), (1,6)\}$ ,  $k(x) = x^2 + 5$ , determine each of the following:

a:  $(f + h)(1) =$

b:  $(k - g)(5) =$

c:  $(f \circ h)(3) =$

d:  $(g \circ k)(7) =$

e:  $f^{-1}(x) =$

f:  $g^{-1}(x) =$

g:  $\frac{1}{f(x)} =$

10. Write the inequality  $|A| < B$  without absolute value bars.

11. Write the inequality  $|A| > B$  without absolute value bars.

12. Solve for  $x$ :  $|2x - 3| < 5$

13. Solve for  $x$ :  $|3x - 2| > 5$

#### IV

1. The period (time for one complete oscillation) of a pendulum is directly proportional to the square root of the length of the pendulum. A pendulum of length 8ft has a period of 2sec. Find a mathematical model expressing the period as a function of the length **and** find the number of swings per second made by a pendulum of 2ft in length.

2. The surface area of a sphere is given by:  $A = 4\pi r^2$ . Suppose a balloon maintains the shape of a sphere as it is being inflated so that the radius is changing at the constant rate of 3 cm per second. If  $f(t)$  centimeters is the radius of the balloon after  $t$  seconds:

a: Compute  $(A \circ f)(t)$  and interpret the result (what does it tell you?)

b: Find the surface area of the balloon after 4 seconds using  $(A \circ f)(t)$ .

\*3. A rectangular field is to be enclosed with 240m of fence but one side of the rectangle is a river so the fencing only needs to be used on the other three sides. Express the area of the field as a function of the length (the dimension parallel to the river), graph the function and give, to the nearest tenth of a meter, the dimensions of the field having the greatest area.

\*4. A manufacturer makes open tin boxes from pieces of tin that are 12cm by 15cm by cutting squares out of the corners and bending up the sides. Find the size, to the nearest .01cm, of the cut-out squares in order that the volume of the boxes is as great as possible.

5. Do you think it is true that  $2^{2n+1} + 1$  is divisible by 3 for all  $n \geq 1$ . If you believe it might be, prove it by induction.

V Simplify:

- |   |   |                            |
|---|---|----------------------------|
| 1. $\frac{\sqrt{x}}{x}$                         | 2. $e^{\ln 3}$                                      | 3. $e^{(1+\ln x)}$         |
| 4. $\ln 1$                                      | 5. $\ln e^7$  | 6. $\log_3(1/3)$           |
| 7. $\log_{1/2} 8$                               | 8. $\ln \frac{1}{2}$                                | 9. $e^{3 \ln x}$           |
| 10. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$ | 11. $27^{2/3}$                                      | 12. $(5a^{2/3})(4a^{3/2})$ |
| 13. $(4a^{5/3})^{3/2}$                          | 14. $\frac{3(n+2)!}{5n!}$ [NOTE: $5n! \neq (5n)!$ ] |                            |

VI

- Write in slope intercept form the line perpendicular to  $2x - 3y = 7$  and passing through  $(5,1)$  by using the **POINT SLOPE** form of a straight line. (look this up if you need to, we will use it!)
- Find the equation of a straight line (in slope intercept form) that is tangent to the circle of radius 2, centered at the origin at a point that is **in** the center of the second quadrant using the **POINT SLOPE** form of a straight line.

VII. Without a calculator, determine the **exact** value of each expression. (Assume principal inverse values). **BE SURE YOU KNOW THESE!**

- |                          |                         |                                   |                                      |
|--------------------------|-------------------------|-----------------------------------|--------------------------------------|
| 1. $\sin 0$              | 2. $\sin \frac{\pi}{2}$ | 3. $\sin \frac{3\pi}{4}$          | 4. $\cos \pi$                        |
| 5. $\cos \frac{7\pi}{6}$ | 6. $\cos \frac{\pi}{3}$ | 7. $\tan \frac{7\pi}{4}$          | 8. $\tan \frac{\pi}{6}$              |
| 9. $\sec \frac{2\pi}{3}$ | 10. $\cos 0$            | 11. $\cos(\sin^{-1} \frac{1}{2})$ | 12. $\sin^{-1}(\sin \frac{7\pi}{6})$ |

VIII Solve for x, where x is a real number. Show the work that leads to your solution.

- |  |   |                           |
|--|---|---------------------------|
| 1. $x^2 + 3x - 4 = 14$                       | 2. $\frac{x^4 - 1}{x^3} = 0$                                | 3. $(x - 5)^2 = 9$        |
| 4. $2x^2 + 5x = 8$                           | 5. $(x + 3)(x - 3) > 0$                                     | 6. $x^2 - 2x - 15 \leq 0$ |
| 7. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$ | 8. $(x - 2)(x + 3)^7(x - 14)^{18}(x + 11)^{29}(x)^{34} > 0$ |                           |
| 9. $27^{2x} = 9^{x-3}$                       | 10. $\log x + \log(x - 3) = 1$                              | 11. $e^{3x} = 5$          |
|  |   | 12. $\ln y = 2x - 3$      |

**IX** State the following formulae: (from memory if at all possible, only look up what you MUST). **Be sure you know these by the first day of class! They will be assumed**

1.  $\sin(A + B) =$
2.  $\cos(A + B) =$
3.  $\sin 2A =$  Show the derivation of this from #1 and/or #2, be CLEAR
4.  $\cos 2A =$  = = (3 forms) Show the derivations from #1 and/or #2
5.  $\sin\left(\frac{1}{2}A\right)$  Show the derivation from #4, be CLEAR
6.  $\cos\left(\frac{1}{2}A\right) =$  Show the derivation from #4, be CLEAR
7.  $\sec^2 A =$  Show the derivation from  $\sin^2 \theta + \cos^2 \theta = 1$ , be CLEAR
8.  $\csc^2 A =$  Show the derivation from  $\sin^2 \theta + \cos^2 \theta = 1$ , be CLEAR
9. In what quadrant is the terminal side of a 100 radian angle that is in standard position? Explain.

**X** Please use the reference on the web site if the Binomial Theorem needs refreshing and email me with questions after you have made a good attempt.

1. Use the binomial theorem to expand  $(a + b)^8$
- \*2. Find the first four terms of  $(2x^2 + y^2)^{12}$  using the **binomial theorem**
- \*3. Find the sixth term of  $(2x - 3)^9$  using the **binomial theorem**
4. Find the coefficient of  $x^6$  in the expansion of  $\left[x^2 - \left(\frac{1}{x}\right)\right]^{12}$  using the **binomial theorem**.
5. Find the constant term of  $(x^2 - 2x^{-2})^{10}$  (NOTE: the constant term of  $x^2 + 2x + 3$  is 3)

**XI** Please use the reference on the web site if Geometric Sequence/Series needs refreshing and email me with questions after you have made a good attempt.

1. Write the first five terms of the geometric sequence in which  $a = -81$  and  $r = 1/3$ .
2. Find the common ratio of a geometric series whose third term is -2 and whose sixth term is 54.
3. Find the sum of the infinite series  $60 - 6 + 0.6 + \dots$

4. Find the sum of the infinite series  $3 + \sqrt{3} + 1 + \dots$
5. Write the rational number  $1.234234\overline{234} \dots$  as a fraction in lowest terms.

**XII Please use the reference on the web site if Polar Coordinates need refreshing and email me with questions after you have made a good attempt. A link is available on the same site from which you downloaded this for POLAR graph paper.**

1. Plot the polar points on a polar coordinate system superimposed on a cartesian coordinate system:

a:  $(1, -\frac{\pi}{4})$       b:  $(3, \frac{5\pi}{6})$       c:  $(-3, \frac{5\pi}{6})$       d:  $(-2, -\frac{\pi}{2})$

2. Write 5 different (non trivial) polar coordinates for the polar point  $(-2, -\frac{5\pi}{4})$
3. Sketch carefully the polar graph of  $r = 2 - 4 \cos \theta$
4. Sketch carefully the polar graph of  $r = 3 + 3 \sin \theta$
5. Sketch carefully the polar graph of  $r = 2 \cos 2\theta$

**XIII Please use the reference on the web site if Parametric Equations need refreshing and email me with questions after you have made a good attempt.**

1. Sketch, on a Cartesian coordinate system, the graph of:  $x = 3 - 2t$ ,  $y = 4 + t$
2. Sketch, on a Cartesian coordinate system, the graph of:  $x = 2t^3$ ,  $y = 4t^2$
3. Sketch, on a Cartesian coordinate system, the graph of:  $x = 9 \cos t$ ,  $y = 4 \sin t$   $t \in [0, 2\pi]$

**XIV Be as analytic as possible but use any method at your disposal as long as it is clear how you are proceeding. Explain yourself carefully and completely. ANSWER TO FOUR DECIMAL PLACES. DO NOT USE TRIAL AND ERROR.**

- \*1. Consider the function  $f(x) = 3x - 4$ . It is clear that  $f(5) = 11$ . What is the greatest distance that  $x$  can be from 5 in order that  $f(x)$  is no farther than 0.06 away from 11?
- \*2. Consider the function  $f(x) = x^2 + 4$ . What is the greatest distance between  $x$  and 2 in order that  $f(x)$  is no farther than 0.1 from 8 as one considers values of  $x$  moving away from  $x=2$ ?

## EVEN AND ODD FUNCTIONS

**Recall:**

**Even functions** are functions that are symmetric over the  $y$ -axis.

To determine algebraically we find out if  $f(x) = f(-x)$

(\*Think about it what happens to the coordinate  $(x, f(x))$  when reflected across the  $y$ -axis\*)

**Odd functions** are functions that are symmetric about the origin.

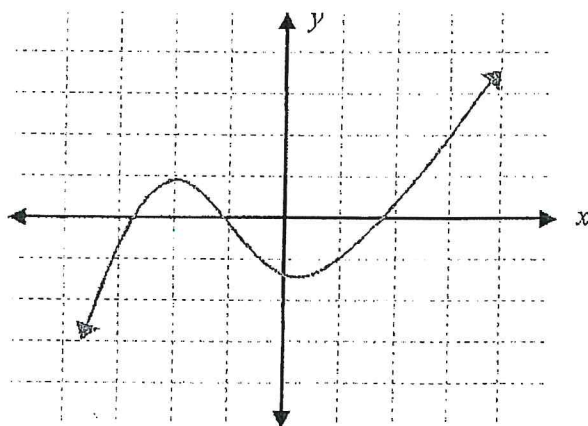
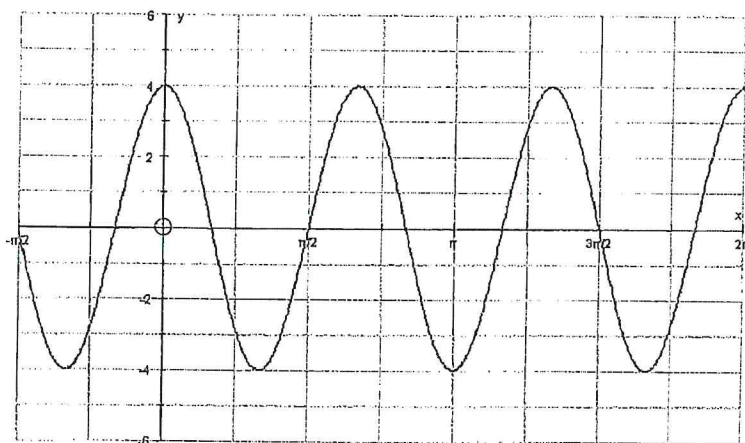
To determine algebraically we find out if  $f(-x) = -f(x)$

(\*Think about it what happens to the coordinate  $(x, f(x))$  when reflected over the origin\*)

State whether the following graphs are even, odd or neither, show ALL work.

77. \_\_\_\_\_

78. \_\_\_\_\_



79. \_\_\_\_\_

$$f(x) = 2x^4 - 5x^2$$

80. \_\_\_\_\_

$$g(x) = x^5 - 3x^3 + x$$

81. \_\_\_\_\_

$$h(x) = 2x^2 - 5x + 3$$

82. \_\_\_\_\_

$$j(x) = 2 \cos x$$

83. \_\_\_\_\_

$$k(x) = \sin x + 4$$

84. \_\_\_\_\_

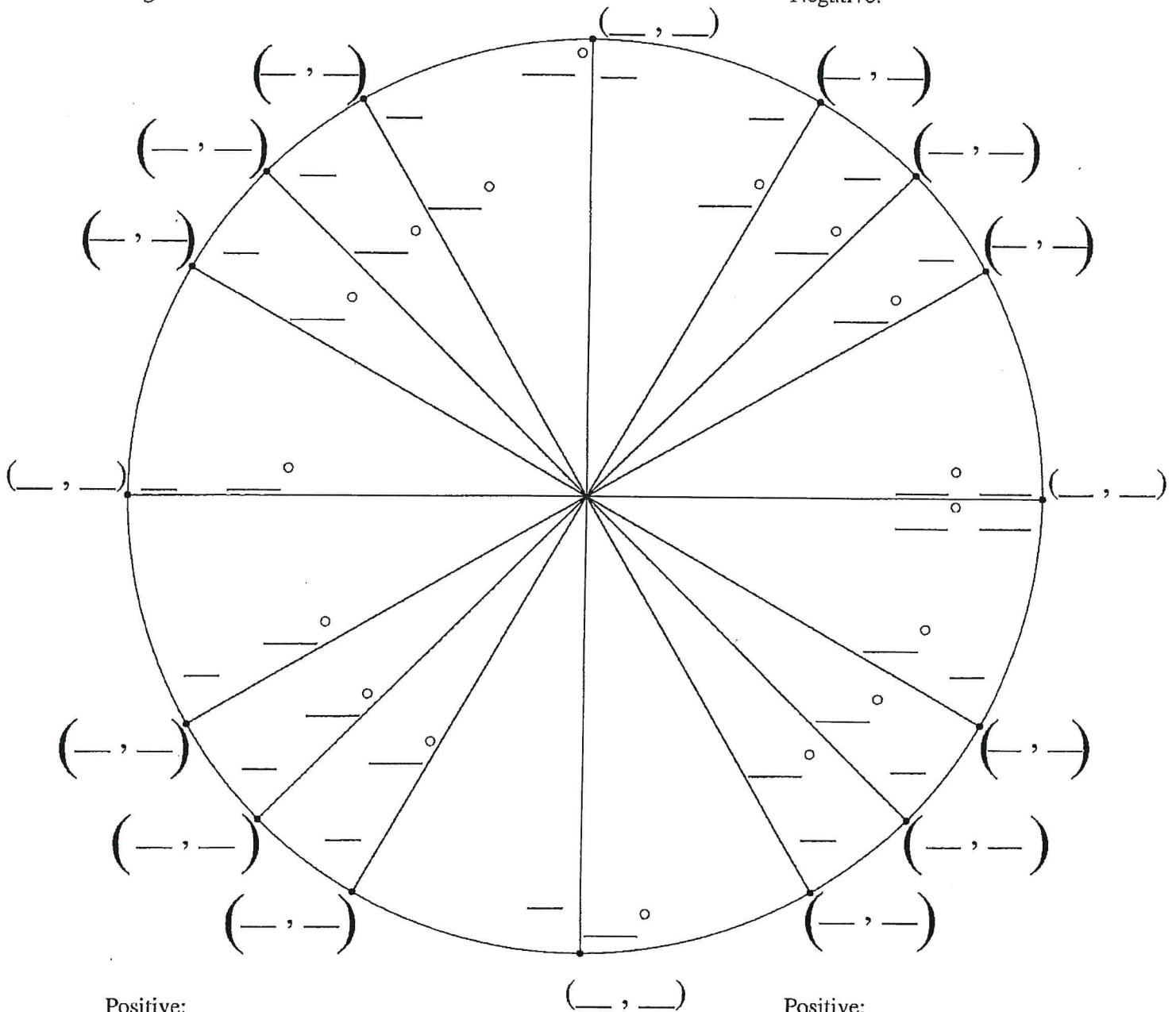
$$l(x) = \cos x - 3$$



# Fill in The Unit Circle

Positive:  
Negative:

Positive:  
Negative:



Positive:  
Negative:

Positive:  
Negative:



## EQUATION OF A LINE

**Slope intercept form:**  $y = mx + b$

**Vertical line:**  $x = c$  (slope is undefined)

**Point-slope form:**  $y - y_1 = m(x - x_1)$

**Horizontal line:**  $y = c$  (slope is 0)

\* LEARN! We will use this formula frequently!

**Example:** Write a linear equation that has a slope of  $\frac{1}{2}$  and passes through the point (2, -6)

**Slope intercept form**

$$y = \frac{1}{2}x + b$$

Plug in  $\frac{1}{2}$  for  $m$

$$-6 = \frac{1}{2}(2) + b$$

Plug in the given ordered

$$b = -7$$

Solve for  $b$

$$y = \frac{1}{2}x - 7$$

**Point-slope form**

$$y + 6 = \frac{1}{2}(x - 2)$$

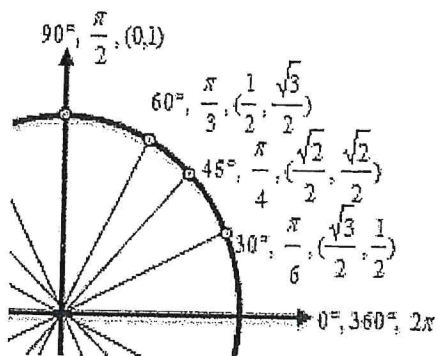
Plug in all variables

$$y = \frac{1}{2}x - 7$$

Solve for  $y$

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.
30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of  $\frac{2}{3}$ .
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line  $y = \frac{5}{6}x - 1$ .
33. Use point-slope form to find a line perpendicular to  $y = -2x + 9$  passing through the point (4, 7).
34. Find the equation of a line passing through the points (-3, 6) and (1, 2).
35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

## UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as  $\sin/\cos$  or the slope of the line.

**Examples:**

$$\sin \frac{\pi}{2} = 1$$

$$\cos \frac{\pi}{2} = 0$$

$$\tan \frac{\pi}{2} = \text{und}$$

**\*You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. a.)  $\sin \pi$

b.)  $\cos \frac{3\pi}{2}$

c.)  $\sin \left( -\frac{\pi}{2} \right)$

d.)  $\sin \left( \frac{5\pi}{4} \right)$

e.)  $\cos \frac{\pi}{4}$

f.)  $\cos(-\pi)$

g.)  $\cos \frac{\pi}{3}$

h.)  $\sin \frac{5\pi}{6}$

i.)  $\cos \frac{2\pi}{3}$

j.)  $\tan \frac{\pi}{4}$

k.)  $\tan \pi$

l.)  $\tan \frac{\pi}{3}$

m.)  $\cos \frac{4\pi}{3}$

n.)  $\sin \frac{11\pi}{6}$

o.)  $\tan \frac{7\pi}{4}$

p.)  $\sin \left( -\frac{\pi}{6} \right)$